Geometry of Economics: Volumetric Distribution Analysis of Economic Continuity and Stability*

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Abstract. While economics derives its value from daily activity of its participants, logically making it a derivative (speed), a traditional approximation of economic variables is carried out using a set of linear and / or nonlinear regression equations and correlation analysis, with no differentiation involved. This explains why the traditional analysis is not capable of identification and prevention of a looming economic crisis: firstly, linear and nonlinear regression value approximation method always relies on continuity assumption of a variable, and secondly, focusing on speed of economics doesn't solve a known limitation of a derivative — its continuity cannot be predicted. This limitation is proposed to be solved with volumetric distribution analysis using volumetric 3D geometry, allowing tracing how distribution of the entire population of the examined variables changes in time and volume as volumetric geometric figures, and what effect it has on continuity of its gradient — the "barometer" of an economic system. Our hypothesis is that a system is stable when it takes a nondegenerate geometric shape and unstable otherwise. An economy can take one shape or another, as volumetric distribution analysis shows, and visualizing it with geometric shapes and respective gradient can help predict its continuity.

Аннотация. В то время как ценность экономики создается за счет повседневной деятельности ее участников, что логически делает ее производной (скоростью), традиционное определение значений экономических переменных осуществляется с помощью набора линейных и/или нелинейных уравнений регрессии и корреляционного анализа без использования дифференциальных уравнений. Это объясняет, почему традиционный анализ не способен к выявлению и предотвращению надвигающегося экономического кризиса: во-первых, метод линейной и нелинейной регрессии всегда опирается на предположение непрерывности переменной, а во-вторых, сосредотачиваясь исключительно на скорости экономики, невозможно решить известное ограничение производной — ее непрерывность не может быть предсказана. Данное ограничение предлагается решить с помощью анализа объемного распределения изучаемых переменных с использованием объемной 3D-геометрии, позволяющей отслеживать изменение распределения совокупности изучаемых переменных во времени и пространстве в виде объемных геометрических фигур, а также влияние, которое она оказывает на постоянство ее градиента — «барометра» стабильности экономической системы. Наша гипотеза заключается в том, что система устойчива, когда она принимает невырожденную геометрическую форму, и нестабильна в обратном случае. Как показывает анализ объемного распределения, экономика может принимать ту или иную форму, и ее визуализация с помощью геометрических фигур и соответствующего градиента поможет предсказать непрерывность ее значений.

Key words: Geometry of economics, volumetric distribution, sustainable economics, economic continuity, income inequality, money paradox.

I.INTRODUCTION

Economics is a mirror of what we do. While economics derives its value from daily activity of its participants, logically making it a derivative (speed), a traditional approximation of economic variables is carried out using a set of linear and/or nonlinear regression equations and correlation analysis, with no differentiation involved. This explains why the traditional analysis is not capable of identification and prevention of a looming economic crisis, because focusing on speed of economics doesn't solve a known limitation of a derivative — its continuity cannot be predicted. This is also true for linear and nonlinear

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regression value approximation — it always relies on continuity assumption of a variable.

Moreover, on the one hand, extrapolation of value derived from a set of its historical values, as the traditional approximation of variables' values method suggests, doesn't take into account a notion of limits that reality may place on a dependent variable. On the other hand, it is known that infinity as a notion can only exist in a theoretical infinite system, but not in a closed system, which is what economic reality is. This mismatch is not only confusing, but it also explains why continuity is assumed in order to justify extrapolation of a dependent variable.

This limitation is proposed to be solved with volumetric distribution analysis using volumetric 3D geometry. We propose visualization of examined variables as volumetric objects, where distribution of one variable is evaluated against another via volumetric visualization over time. Therefore it is possible to trace how distribution of the entire population of the examined variables changes in time and volume as volumetric geometric figures, and what effect it has on continuity of its gradient — the "barometer" of an economic system.

The idea behind volumetric distribution analysis was to develop the methodology of creating accurate models of economic systems, correctly identify principal trends, forecast their future development and help identify actions and points of their application in order to ensure their continuity.

Our hypothesis is that a system is stable when it takes a nondegenerate geometric shape and unstable otherwise. For example, elliptical (2D) or ellipsoid (3D) distribution of variables, approximating multivariate normal distribution of a closed system, represents a more stable/continuous system due to more centered distribution around the mean, while transformation to a hyperbolic (2D) or hyperboloid (3D) distribution would cause instability due to its degenerescence.

In addition, evaluation of the dynamics of change in volume of formed volumetric geometric figures allows us to see not only the transformation of the tested variables in time, but also an onset of critical trends that jeopardize the continuity of the examined variables and the system as a whole.

II. PROBLEM

We will exemplify the problem with two variables — money and number of people. Money as means of exchange reflects economic activity, and stability of the examined system will depend on respective distribution of money between the market participants throughout time.

A traditional analysis of money and number of people distribution is recorded using an exponential distribution function obtained from non-linear regression analysis, where expected value of an exponential random variable X is dependent solely on λ (constant of proportionality, or rate of occurrence):

$$E\,[X] = 1/\lambda$$

Thus, as λ approaches zero, the value of exponential random variable X approaches infinity.

While an infinite number of people or money is not feasible for practical reasons, we will use the following model with preset boundaries in order to formulate the relationship between the examined variables:

Let's consider the time series:

$$0 < t_1 < t_2 < \ldots < t_{n-1} < T$$

Assume that at the moment of time t_i the wealth value w_i of each market participant (person) is changing due to market activity (e.g. profit or loss) in proportion of the total common wealth (total amount of money) W_i and the probability k_1 to make the profit from the market, where the probability depends on the market participant's current wealth w_i:

$$p_{i+1} = k_1 \cdot W_i$$

The market participant pays taxes to the common wealth at a rate k_2:

$$x_{i+1} = k_2 \cdot p_{i+1}$$

The market participant returns expenses to the common wealth at a rate k_3:

$$e_{i+1} = k_3 \cdot w_i$$

Therefore the total balance of each individual at each time step equals to:

$$w_{i+1} = w_i + p_{i+1} - x_{i+1} - e_{i+1}$$

In an environment where the number of market participants and total common wealth are known, the above model may become unstable at a certain point in time due to the following reasons:

- Common wealth is constant in value, but its distribution over time degenerates following power law distribution rule, stifling future activity (for example, most of the wealth becomes controlled by very few participants who become unwilling to participate in future activity), and

• External factor (s) affecting ability or willingness of the players to participate.

Such a scenario is best visualized when variance of the probability density function approaches zero, while the function itself will approach the delta function:

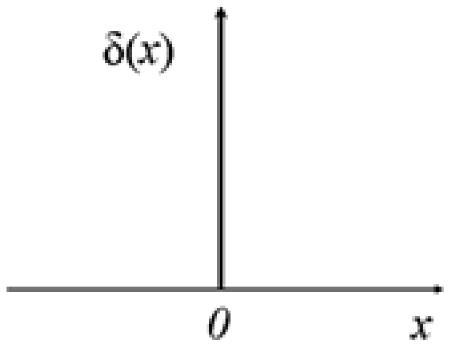

Figure 1.

$$\int_{-\infty}^{+\infty} \delta_{(x)} dx = 1 \text{ and } \int_{-\infty}^{+\infty} \delta_{(x-\alpha)} dx = 1$$

Therefore, regardless of a certain speed of economy prior to probability density function becoming zero, the system still collapses, but we cannot predict the critical point of such an occurrence using traditional tools and techniques of regression equations and correlation analysis.

III. MODEL

We built a 3D model of market participants and respective money distribution. Each market participant is defined as a unit of volume ($Vp = 1$). Each unit is also assigned a certain value based on the amount of money in possession. Each unit is then located at a level appropriate to its value.

Should such allocation follow the power law distribution, resulting in the majority of people having less money and *vice versa*, we will see a distribution that can be roughly approximated by a volumetric pyramid, with money (value) disproportionately increasing at the top of the pyramid:

As follows from the power law distribution, top one percent of people possess a disproportionately larger amount of common wealth than those at the bottom. Further, about 25% of people possess about 75% of common wealth. The slope of an edge of the triangle/pyramid will get steeper as wealth distribution gets more uneven. While our approach is very simplistic, should the shape of the uneven distribution be hyperbolic, not pyramidal as we have shown above, we can take an average slope of the surface between the peak and the flatter regions and that will still result in a pyramidal or conical shape (shape of the base is not relevant to this discussion).

Conversely, should the wealth distribution between the market participants be more uniform and centered around the median of the population, the resulting shape would be an ellipsoid as shown below.

Our hypothesis is that ellipsoid type distribution makes economic activity of system more stable and thus more continuous than a pyramid or hyperboloid due to a more uniform (normal) distribution. For example, a society, where the middle class possesses the majority of wealth (e.g. U.S. in 1960-s), would show an ellipsoid wealth distribution, making such a society financially stable and generally happy with the state of its economics.

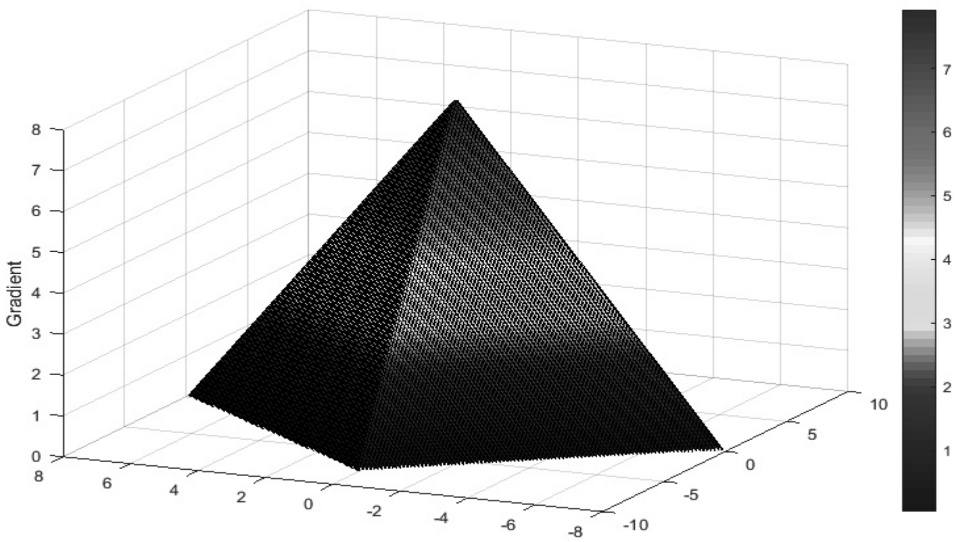

Figure 2.

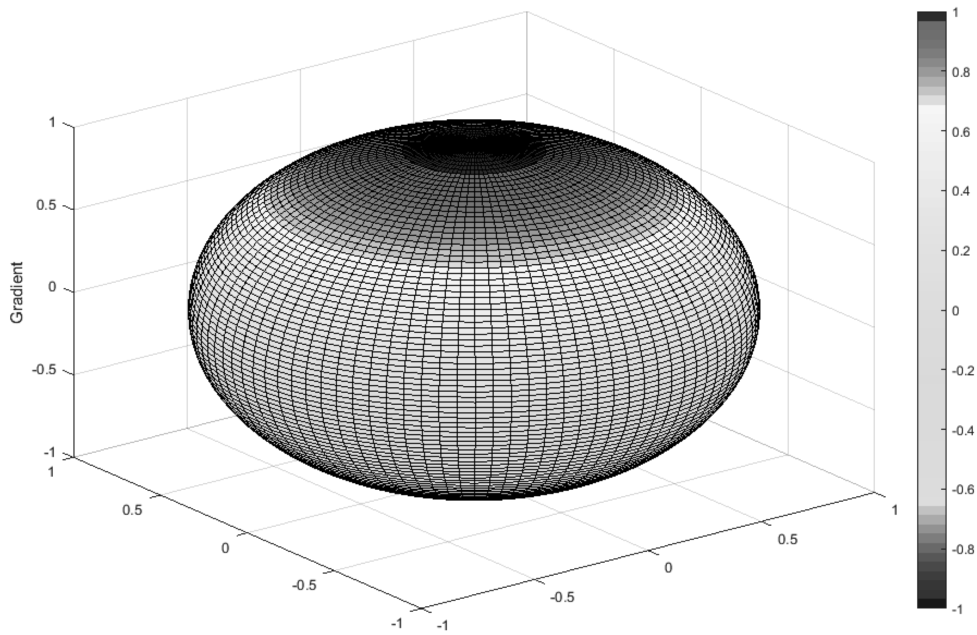

Figure 3.

The shape of wealth distribution figure can be convex or concave. Should the surface of the figure become concave, i.e. with a smaller number of people breaking asymptotically farther away from the mean, it will mean that the economic system as a whole is becoming less stable, thus putting its continuity at risk, and *vice versa* — should the surface of the figure become convex, it will mean that the economic system shows more stability, and thus its continuity will be of less concern.

As a practical example of how we can represent a system geometrically, let's consider the volume of a pyramid with a square base, with height h and angle α (angle between one of its non-horizontal edges and the base plane) as follows:

$$V = \frac{2h^3 cot^2(\alpha)}{3}$$

Further, each participant $\boldsymbol{n}$ occupies a unit of volume. Therefore, if we know the total number of market participants (e.g. $\boldsymbol{n} = 100$), then we know the volume of the shape: $\boldsymbol{V} = 100$. The number of layers making up the height (levels) of the pyramid h will depend on the disparity of wealth distribution between levels — its height. If there are 10 distinct levels of magnitude, then $\boldsymbol{h} = 10$. The angle α is calculated using the following formula, showing interdependence between $\boldsymbol{h}$ and α:

$$\alpha = \text{arccot}\sqrt{\frac{3V}{2h^3}} = \text{arccot}\sqrt{\frac{3}{20}} = \text{arccot}0.3873 \approx 69°$$

However, if V = 100 (as before) and h = 3, then:

$$\alpha = \text{arccot}\sqrt{\frac{50}{9}} \approx 23°$$

Height $\boldsymbol{h}$ can also be derived from known values of volume $\boldsymbol{V}$ and angle α as follows:

$$h = \left(\frac{3V}{2cot^2\alpha}\right)^{1/3}$$

The latter equation will be the link between the vector fields (angle) and scalar fields (height) as matrix calculus equations allow derivation of scalar fields form vector fields, thus turning direction into magnitude. $\boldsymbol{\Phi A}$ scalar field will have some scalar value $\boldsymbol{A}$ at every point in space, and can be represented by (in 3 dimensions):

$$A = f(x, y, z)$$

Where $\boldsymbol{f(x, y, z)}$ will demonstrate respective scalar value $\boldsymbol{A}$ of a unit of volume $\boldsymbol{Vp}$. The rate of change of this field with respect to time at any point is given by:

$$\frac{dA}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$$

For a vector field, there will be some vector value $\boldsymbol{v}$ at every point in space, then each component of the vector will be a function of x, y and z independently, so:

$$v = [V1, V2, V3] = [f(x,y,z), g(x,y,z), h(x,y,z)]$$

Then for the rate of change of the vector field at any point, we will have a partial differential equation for each component (3 in this case), for example:

$$\frac{dV1}{dt} = f^{'}(x) + f^{'}(y) + f^{'}(z)$$

By analyzing vector fields and their characteristics, which in turn result in changes to scalar fields and respective divergence of the surface of the figure, we can further analyze how the system changes its shape and if that change adds to its stability and continuity, or not.

The amount of money and its infusion into a financial system is a whole different topic altogether; however, it also has a direct connection to respective distribution for the purpose of continuity of the overall economic system. If one of the variables is artificially altered incorrectly as an attempt to benefit the overall system, then such a system can still run a risk of instability, and vice versa.

For practical reasons, if we equalize energy and activity $\boldsymbol{A}$, we will get that in order to maintain a significant level of activity, or money circulation, we must have a continuous source of money in case of system instability, but the magnitude of activity obtained by infusion of new money into the system may still decrease inversely to profitability $\boldsymbol{P}$ perceived within the system:

$$A = \frac{1}{P}$$

Such inverted relation between business activity and profitability in a closed system could add to an even higher disparity of wealth distribution, making such system even less stable. Tracking distribution of new money with volumetric distribution analysis will help better understand the dynamics and identify stress points early in the game and act accordingly in order to ensure the overall system's continuity.

IV. CONCLUSIONS

An economy can take one shape or another, as volumetric distribution analysis shows, and visualizing it with geometric shapes and gradient can help predict its continuity. Analysis of the vector fields and respective scalar fields, which can be represented as volumetric shapes of the overall distribution of value between examined variables, provides a useful tool for assessment of an economic system's stability.

A continuous, or stable, economy would exist if there was an equal contribution by all participants involved in the economic process — someone creates something and gets something else in exchange. Should there be shortage or excess of contribution or demand, respective distribution and gradient will immediately reflect the distortion, which we will be able to see through respective change of geometric shapes and scalar fields of the examined variables.

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